Boosting Automated Reasoning using Machine Learning

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A Wake-up Picture

Artificial Intelligence
$\rightarrow$ **Automated Reasoning**

Gottfried Leibniz's dream: *Calculus ratiocinator*

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Symbolic, Logical Calculi, "Sound and Complete", Undecidable!, . . .

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(as opposed to, e.g., the Interactive Theorem Proving)

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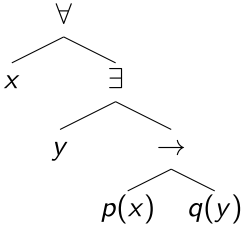
$\rightarrow$ **Saturation-based ATPs (for FO logic)**

The Flavours of ML in Automated Theorem Proving

Our basic data structures are (primarily) logical formulas

- symbolic expression: "$\forall x \exists y . p(x) \rightarrow q(y)$"
- in fact, a tree-like object:



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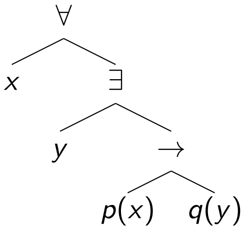

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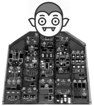
How to apply modern ML to this?

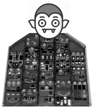
- hand-crafted features
- recursive neural networks
- graph convolutional networks
- ...



Vampire

- Automatic Theorem Prover (ATP) for First-order Logic (FOL) with equality and theories
- state-of-the-art saturation-based prover

Deepire: Powering ATPs using Neural Networks



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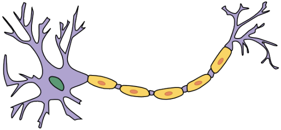
Neural (internal) guidance

- targeting the clause selection decision point
- supervised learning from successful prover runs



Outline

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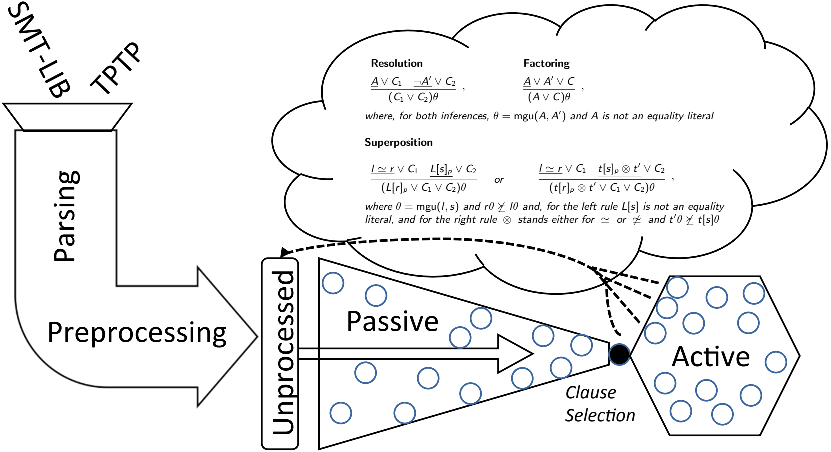
Saturation-based Theorem Proving in One Slide



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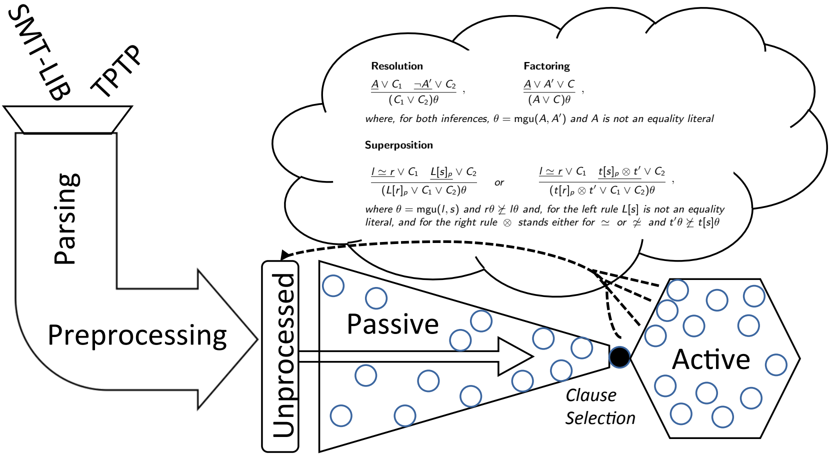

At a typical successful end: $|Passive| \gg |Active| \gg |Proof|$

Clause selection: traditionally and neurally

Traditionally: simple clause evaluation criteria

- weight: prefer clauses with fewer symbols
- age: prefer clauses that were generated long time ago
- ...

Combine these using priority queues into a single scheme

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How to improve this with ML?

- train a classifier for recognizing clauses that appeared in past proofs (as opposed to those selected, but not found useful)
- integrate into the selection mechanism,
  prioritizing clauses classified positively

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Started off with ENIGMA:

- ENIGMA: Efficient Learning-Based Inference Guiding Machine [Jakubův&Urban,2017]
- ENIGMA-NG: Efficient Neural and Gradient-Boosted Inference Guidance for E [Chvalovský et al.,2019]
- ENIGMA Anonymous: Symbol-Independent Inference Guiding Machine [Jakubův et al.,2020]

See also:

- Deep Network Guided Proof Search [Loos et al.,2017]
- Property Invariant Embedding for Automated Reasoning [Olšák et al.,2020]

Machine Learning Guided Clause Selection

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Most recently also Deepire:

- New Techniques that Improve ENIGMA-style Clause Selection Guidance (submitted to CADE)
- Vampire With a Brain Is a Good ITP Hammer (submitted to ITP)

Similar to ENIGMA:

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- good are those that appeared in past proofs

Deepire: Neurally Guided Clause Selection in Vampire

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- Learn from clause derivation trees only
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- Integrate using layered clause queues
  A smooth improvement of the base clause selection strategy.
- Tree Neural Networks: constant work per derived clause
- A signature agnostic approach
- Lazy evaluation trick (not all derived need to be evaluated)

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Preliminary Evaluation on Mizar "57880"

- Learn from 63595 proofs of 23071 problems (three 30s runs)
- Deepire solves 26217 (i.e. +4054) problems in a single 10s run

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Things to consider:

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(Similarly: a fixed number of rounds of message passing in a GCN for an arbitrary formula also does not "feel right".)

Even Higher-level

Relation to AGI?

Logic as a Means to Explainable AI?

Embeddings Respecting Semantic Logical Relations?

One More Picture

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Thank you for attention!